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# SOME MODELS FOR RAINFALL

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Models are given both for the probability of daily precipitation and for the probability of an "Extreme Value" for daily precipitation during a given month for each of ten widely distributed stations. Two sets of models are given in each case. Each model in the first set is valid only for a specific month. The models in the second set are applicable to any month of the year. A measure of goodness is given. For daily precipitation, the models are the lognormal and the modified kappa. For extreme values of daily precipitation, the "Extreme Value" model is used.

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## INTRODUCTION

There are many situations where it is useful to know the probability that some weather event will occur on some specified date in the future. One way to estimate this probability is to examine past records, and find the proportion of time that the event occurred in previous years. For example, if one were interested in the probability of rain in Honolulu on May 8, 1982, one method would be to examine the available records of Honolulu for May 8, for previous years. Presently, large environmental data bases exist which can be used to estimate the probabilities of various weather occurrences. Because of the massive amount of data, summaries are usally utilized. USAFETAC, Air Weather Service regularly produces, for a large number of stations, a "Revised Uniform Summary of Surface Weather Observations." These "RUSSWO's" provide a very convenient summary and are widely utilized.

There are a number of situations where even these summaries are unwieldly. For example, decision models are now widely used in a variety of disciplines. In a number of these, environmental conditions are required as inputs. To input raw historical data for a decision model into a computer may require a prohibitive amount of storage space. If one can give frequency distributions for the states of a given weather element, then it may be possible to substitute models (formulas) for a summary. This would substantially increase the number of situations where it would be possible to utilize decision models requiring environmental inputs.

Part I of this report gives statistical models for the probability of daily precipitation amounts. Part II gives models for the probability of "extreme values" for precipitation during a given month. In all cases the models are designed to answer questions requiring as answers, estimation of the cumulative distribution function. This report gives models for ten stations at diverse world-wide locations. In addition to separate models for each month, additional models are given for which month of the year is an input to a general model, valid for all months. For the latter model if one wished results for May, for example, we would set M=5, and similarily M=11 for November. As would be expected, the general model does not give results as accurate as the individual models. An estimate of the accuracy of the formulas is given in all cases. The models were developed from data obtained from a "Revised Uniform Summary of Surface Weather" for the appropriate station, published by USAFETAC, Air Weather Service.

## PART I DAILY PRECIPITATION

### 1. The Lognormal Model For Daily Precipitation

Let the proportion of days when there is no rain be denoted by D, and let F(x) be the probability of an amount of rain less than or equal to x. Assume for the days when rain occurs, that the amount of rain has a lognormal distribution. Then, if $\Phi(z)$ is the cumulative normal distribution, we may write

$$F(x) = D + (1-D)\Phi(z)$$

where $z = (\ell n\ x-\mu)/\sigma$, and $\mu$ and $\sigma$ are the mean and standard deviation of $\ell n\ x$.

For any specified month, the lognormal model, denoted by MODEL I, was found to fit the data quite well for each station for each month. Two different methods were used to estimate the parameters $\mu$, $\sigma$ and D. The first and simplest was to estimate D as the proportion of days when no rain occurred. Estimates of $\mu$ and $\sigma$ were then obtained using the method of maximum likelihood. The maximum likelihood estimates of $\mu$ and $\sigma$ are the arithmetic mean and the standard deviation (with divisor the sample size n instead of n-1) of the natural logarithms of the data for days when there was rain.

The second method obtained estimates of $\mu$, $\sigma$ and D using least squares non-linear regression techniques. Let y = C(x) be the proportion of the days when the rainfall is x inches or less. Then we regress y on $F(x) = D + (1-D)\Phi(z)$. The statistical package SAS has a non-linear regression procedure called NONLIN, and this was used to obtain estimates m, s and d for $\mu$, $\sigma$ and D. This technique minimizes the sum of squares between the empirical cumulative distribution function and the model cumulative distribution function at the boundary points of the x-intervals.

Since we are in general interested in estimating the probabilities of the occurrences of rainfall less than (greater than) some specified amount x, and not merely the estimates of $\mu$, $\sigma$ and D, the second procedure seems the more appropriate. Section 5 gives the estimates m, s and d from the second method for each of the stations for each month of the year.

Model I results in separate values of m, s, and d for each month of the year. Models II and III are comprehensive models for a given station. The twelve monthly values for each of m, s and d in Model II are represented by a cubic polynomial in M, the month of the year (M=1 for January, M=2 for February, etc.).

Model III was obtained by examining the residuals from Model II and "fitting" the residuals to an appropriate sine term. The twelve monthly values for each of m, s and d in Model III are thus represented by a cubic polynomial in M plus a sine term.

The polynomials for each of Models II and III are given in Section 5.

## 2. The Kappa and the Modified Kappa Model For Daily Precipitation

Mielke (1973) has recently introduced a family of frequency distributions. This family has been termed the Kappa distribution and its cumulative distribution function is given by

$$F(x) = [(x/\beta)^{\alpha\theta}/(\alpha + (x/\beta)^{\alpha\theta})]^{1/\alpha\theta} \qquad \alpha,\beta,\theta > 0.$$

For precipitation, Mielke used $\theta = 1$, and termed the resulting family the "two parameter Kappa distribution". For the data from the stations studied in this report, both the "three parameter Kappa model" and the "modified three parameter model" gave fits which were better than those for the "two parameter" family. The "modified Kappa model" has a cumulative distribution given by

$$F(x) = [(x/\beta)^{\alpha\theta}/(\alpha + (x/\beta)^{\alpha\theta})]^{1/\alpha} \qquad \alpha,\beta,\theta > 0.$$

Essenwanger (1976) has given the maximum likelihood estimates for $\alpha$, $\beta$ and $\theta$ for the Kappa model. However, for our modeling purposes we prefer the estimates obtained by the "least squares" method. Using this method, the estimates a, b, t for $\alpha$, $\beta$ and $\theta$ are those values which minimize the sum of squares between the observed empirical cumulative distribution function and the model cumulative distribution function. The empirical cumulative distribution function was obtained from the appropriate RUSSWO's. The categories of "none" and "trace" were combined to give "no precipitation". "No precipitation" is taken to mean precipitation less than or equal to .005 inches.

Two procedures were used to obtain the "least squares estimates". One was non linear regression of the observed empirical cumulative distribution values on the model cumulative distribution function. The SAS (Statistical Analysis System) procedure NONLIN was used. An equivalent method was to use an unsophisticated "grid search". Using the data from the RUSSWO's, the sum of squares of the differences between the empirical cumulative distribution function and the model cumulative distribution function was calculated for a grid of values for $\alpha$, $\beta$ and $\theta$. The "least squares" estimates a, b, t are those values of $\alpha$, $\beta$ and $\theta$ in the grid for which the above sum of squares of differences was smallest.

In Section 5, we give the values a, b, t for the "Modified Kappa" model. We term this Model IV. A comprehensive model for each station was also developed using the "Modified Kappa" model. In Model V, the values a, b, t are given as quadratic polynomials in M, the month of the year. The polynomials for Model V are given in Section 5.

It should be noted that to obtain the estimated probability of no precipitation (including trace) using the modified Kappa model, we put x = .005 inches.

## 3. Goodness of Fit of The Lognormal and Modified Kappa Models

It is expected that the models will be used for estimating the probabilities of rainfall less than (greater than) a specified amount. Thus the measure of "goodness" used was the root mean square of the difference between the empirical cumulative distribution function and the model cumulative distribution function, summed over all the boundary points between categories in the table of observed values in the RUSSWO's.

As previously mentioned, the three parameter modified Kappa distribution gave decidedly better fits than the two parameter kappa distribution. The three parameter modified Kappa gives, on the average, slightly better fits than the lognormal distribution. An advantage of either Kappa distribution is that no tables are needed to calculate needed probabilities since each is a closed form distribution.

Section 5 gives the RMS of the fits for all of the lognormal and modified Kappa distributions.

## 4. Use of the Models

Suppose we wish to estimate the probability of more than 0.5 inch of rain on June 20, 1984, in Bedford, Massachussetts. We first find the probability of 0.5 inch or less.

### Lognormal model for June

Putting x = .5, we have

$$z = (\ell n\ x-\mu)/\sigma$$

$$\doteq (-.693--3.336)/2.230$$

$$= 1.185$$

then the probability of 0.5 inch or less is estimated by

$$D + (1-D)\ \Phi\ (z) \doteq .532 + (1-.532)\ \Phi\ (1.185)$$

$$= .945\ .$$

The probability of more than 0.5 inches is 1-.945 = .055.

### Kappa model for June

Putting x = .5, and using a = 70, b = .450 and t = .08

$$F(x) = [(.5/.450)^{5.6} / (70 + (.5/.450)^{5.6})]^{1/70}$$

$$= .949$$

The probability of more than 0.5 inches is estimated to be .051.

### Lognormal models (overall - Model II)

From Section 5, using M = 6, we calculate

$$d = .504,\quad m = -3.923,\quad s = 2.314.$$

Then $z = (-.693--3.923)/2.314$

$$= 1.396.$$

Estimated probability of less than .5 inches is estimated by

$$.504 + (1-.504)\ \Phi\ (1.396) = .960.$$

The probability of more than .5 inches is estimated to be .040.

5. Tables of Models for Daily Precipitation

Station 12867 Patrick AFB

Model I (Lognormal)

| | d | m | s | RMS of Fit |
|---|---|---|---|---|
| Jan. | .663 | -3.757 | 2.321 | .022 |
| Feb. | .618 | -3.233 | 2.370 | .024 |
| Mar. | .649 | -3.111 | 2.326 | .018 |
| Apr. | .714 | -3.445 | 2.392 | .018 |
| May | .611 | -3.130 | 2.383 | .023 |
| Jun. | .494 | -2.620 | 2.318 | .025 |
| Jul. | .527 | -2.845 | 2.295 | .026 |
| Aug. | .504 | -2.915 | 2.304 | .022 |
| Sep. | .384 | -2.675 | 2.341 | .028 |
| Oct. | .466 | -3.046 | 2.351 | .023 |
| Nov. | .632 | -3.743 | 2.116 | .018 |
| Dec. | .656 | -3.954 | 2.201 | .021 |

Model II (Lognormal)

$$d = .557 + .105M - .027M^2 + .002M^3$$
$$m = -3.712 + .075M - .031M^2 - .003M^3$$
$$s = 2.314 + .021M - .002M^2$$

RMS Error = .128

Model III (Lognormal)

$$d = .638 + .054M - .019M^2 + .001M^3 - .050 \sin(\pi(M-.5)/2)$$
$$m = -3.950 + .224M + .005M^2 - .002M^3 + .145 \sin(\pi(M-.5)/2)$$
$$s = 2.309 + .023M - .003M^2 - .026 \sin(\pi(M-1.5)/2)$$

RMS Error = .033

Station 12867 Patrick AFB (CONT.)

Model IV (Modified Kappa)

| | a | b | t | RMS of Fit |
|---|---|---|---|---|
| Jan. | 40 | .100 | .05 | .015 |
| Feb. | 110 | .975 | .05 | .011 |
| Mar. | 100 | .800 | .05 | .009 |
| Apr. | 30 | .100 | .05 | .017 |
| May | 110 | .975 | .05 | .010 |
| Jun. | 100 | 1.325 | .08 | .013 |
| Jul. | 80 | .800 | .08 | .016 |
| Aug. | 110 | 1.150 | .08 | .008 |
| Sep. | 40 | .975 | .11 | .010 |
| Oct. | 60 | .800 | .08 | .008 |
| Nov. | 70 | .275 | .05 | .006 |
| Dec. | 80 | .275 | .05 | .007 |

Model V (Modified Kappa)

$a = 61.59 + 7.880M - .652M^2$

$b = .0095 + .361M - .029M^2$

$t = .021 + .015M - .001M^2$

RMS Error = 0.036

Station 13802 Scott AFB

Model I (Lognormal)

| | d | m | s | RMS of Fit |
|---|---|---|---|---|
| Jan. | .520 | -4.016 | 2.198 | .031 |
| Feb. | .489 | -3.825 | 2.275 | .004 |
| Mar. | .466 | -3.504 | 2.272 | .003 |
| Apr. | .467 | -3.078 | 2.341 | .036 |
| May | .517 | -2.970 | 2.387 | .029 |
| Jun. | .562 | -2.924 | 2.370 | .024 |
| Jul. | .613 | -2.994 | 2.377 | .023 |
| Aug. | .663 | -3.120 | 2.490 | .021 |
| Sep. | .658 | -3.084 | 2.403 | .020 |
| Oct. | .667 | -3.080 | 2.273 | .017 |
| Nov. | .602 | -3.344 | 2.303 | .025 |
| Dec. | .526 | -3.661 | 2.295 | .003 |

Model II (Lognormal)

$$d = .610 - .116M + .027M^2 - .002M^3$$
$$m = -4.465 + .444M - .035M^2$$
$$s = 2.113 + .074M - .004M^2$$

RMS Error = .121

Model III (Lognormal)

$$d = .610 - .116M + .027M^2 - .002M^3$$
$$m = -4.519 + .491M - .043M^2 + .001M^3 - .093 \sin(\pi(M-1.5)/2.5)$$
$$s = 2.108 + .075M - .005M^2 - .027 \sin(\pi(M-1.5)/2)$$

RMS Error = .030

Station 13802 Scott AFB (CONT.)

Model IV (Modified Kappa)

| | a | b | t | RMS of Fit |
|---|---|---|---|---|
| Jan. | 110 | .625 | .05 | .010 |
| Feb. | 100 | .975 | .05 | .018 |
| Mar. | 80 | .625 | .08 | .011 |
| Apr. | 100 | 1.150 | .08 | .021 |
| May | 120 | .975 | .08 | .016 |
| Jun. | 40 | .450 | .08 | .018 |
| Jul. | 120 | 1.325 | .05 | .015 |
| Aug. | 90 | .625 | .05 | .013 |
| Sep. | 120 | .975 | .05 | .010 |
| Oct. | 90 | .625 | .05 | .009 |
| Nov. | 120 | 1.150 | .05 | .013 |
| Dec. | 110 | .975 | .05 | .014 |

Model V (Modified Kappa)

$$a = 110. - 6.409M + .584M^2$$
$$b = .756 + .023M - .001M^2$$
$$t = .053 + .006M - .001M^2$$

RMS Error = 0.024

Station 14601 Bangor, Maine

Model I (Lognormal)

| | d | m | s | RMS of Fit |
|---|---|---|---|---|
| Jan. | .436 | -3.135 | 2.259 | .037 |
| Feb. | .436 | -3.158 | 2.265 | .034 |
| Mar. | .448 | -3.274 | 2.227 | .032 |
| Apr. | .443 | -3.312 | 2.158 | .031 |
| May | .450 | -3.349 | 2.196 | .030 |
| Jun. | .463 | -3.370 | 2.269 | .032 |
| Jul. | .476 | -3.390 | 2.723 | .028 |
| Aug. | .516 | -3.473 | 2.172 | .024 |
| Sep. | .530 | -3.203 | 2.295 | .026 |
| Oct. | .520 | -3.060 | 2.272 | .022 |
| Nov. | .416 | -2.969 | 2.399 | .040 |
| Dec. | .425 | -3.353 | 2.224 | .027 |

Model II (Lognormal)

$$d = .487 - .052M + .013M^2 - .001M^3$$

$$m = -2.834 - .238M + .032M^2 - .001M^3$$

$$s = 2.354 - .093M + .016M^2 - .001M^3$$

RMS Error = .155

Model III (Lognormal)

$$d = .476 - .045M + .011M^2 - .001M^3 + .020 \text{ Sin } (\pi(M-1.5)/3)$$

$$m = 2.873 - .210M + .026M^2 - .001M^3 + .073 \text{ Sin } (\pi(M-1.5)/3)$$

$$s = 2.364 - .097M + .017M^2 - .001M^3 + .049 \text{ Sin } (\pi(M-1.5)/2)$$

RMS Error = .034

Station 14601 Bangor, Maine (CONT.)

Model IV (Modified Kappa)

| | a | b | t | RMS of Fit |
|---|---|---|---|---|
| Jan. | 40 | .450 | .11 | .024 |
| Feb. | 40 | .450 | .11 | .022 |
| Mar. | 110 | .975 | .08 | .017 |
| Apr. | 50 | .450 | .11 | .015 |
| May | 90 | .800 | .08 | .016 |
| Jun. | 100 | .800 | .08 | .013 |
| Jul. | 50 | .450 | .08 | .007 |
| Aug. | 40 | .275 | .08 | .009 |
| Sep. | 90 | .625 | .08 | .014 |
| Oct. | 40 | .450 | .08 | .010 |
| Nov. | 90 | 1.675 | .08 | .025 |
| Dec. | 110 | .975 | .08 | .014 |

Model V (Modified Kappa)

$$a = 56.136 + 1.741M + .062M^2$$

$$b = .724 - .087M + .010M^2$$

$$t = .119 - .009M + .0005M^2$$

RMS Error = 0.020

Station 14702 Bedford, Massachussets

Model I (Lognormal)

| | d | m | s | RMS of Fit |
|---|---|---|---|---|
| Jan. | .467 | -3.097 | 2.322 | .031 |
| Feb. | .450 | -3.214 | 2.271 | .030 |
| Mar. | .487 | -3.129 | 2.337 | .032 |
| Apr. | .454 | -3.052 | 2.271 | .032 |
| May | .487 | -3.159 | 2.233 | .029 |
| Jun. | .532 | -3.336 | 2.230 | .026 |
| Jul. | .542 | -3.461 | 2.308 | .027 |
| Aug. | .570 | -3.301 | 2.354 | .025 |
| Sep. | .591 | -3.305 | 2.323 | .020 |
| Oct. | .604 | -3.057 | 2.320 | .019 |
| Nov. | .492 | -3.104 | 2.469 | .032 |
| Dec. | .488 | -3.241 | 2.371 | .030 |

Model II (Lognormal)

$$d = .516 - .062M + .016M^2 - .001M^3$$
$$m = -3.101 - .034M + .001M^2$$
$$s = 2.356 - .043M + .006M^2$$

RMS Error = .140

Model III (Lognormal)

$$d = .531 - .072M + .018M^2 - .001M^3 - .011 \text{ Sin } (\pi(M- .5)/1.5)$$
$$m = -2.956 - .121M + .016M^2 - .001M^3 - .077 \text{ Sin } (\pi(M- .5)/3)$$
$$s = 2.431 - .090M + .014M^2 - .001M^3 + .046 \text{ Sin } (\pi(M-2.5)/2)$$

RMS Error = .030

Station 14702 Bedford, Massachussets (CONT.)

Model IV (Modified Kappa)

| | a | b | t | RMS of Fit |
|---|---|---|---|---|
| Jan. | 120 | 1.150 | .08 | .017 |
| Feb. | 90 | .975 | .08 | .014 |
| Mar. | 100 | .975 | .08 | .014 |
| Apr. | 120 | 1.325 | .08 | .021 |
| May | 90 | .975 | .08 | .015 |
| Jun. | 70 | .450 | .08 | .013 |
| Jul. | 120 | 1.325 | .05 | .015 |
| Aug. | 30 | .275 | .08 | .018 |
| Sep. | 110 | .975 | .05 | .011 |
| Oct. | 120 | 1.325 | .05 | .013 |
| Nov. | 90 | .975 | .08 | .017 |
| Dec. | 90 | .800 | .08 | .013 |

Model V (Modified Kappa)

$$a = 129.0 - 13.045M + .894M^2$$

$$b = 1.327 - .145M + .010M^2$$

$$t = .093 - .006M + .0004M^2$$

RMS Error = 0.020

Station 22521 Honolulu, Hawaii

Model I (Lognormal)

| | d | m | s | RMS of Fit |
|---|---|---|---|---|
| Jan. | .448 | -3.830 | 2.315 | .034 |
| Feb. | .410 | -4.097 | 2.103 | .032 |
| Mar. | .385 | -4.385 | 2.094 | .044 |
| Apr. | .342 | -4.729 | 1.837 | .044 |
| May | .385 | -5.034 | 1.637 | .047 |
| Jun. | .347 | -5.267 | 1.279 | .042 |
| Jul. | .355 | -5.194 | 1.342 | .042 |
| Aug. | .340 | -5.070 | 1.491 | .005 |
| Sep. | .411 | -4.934 | 1.590 | .040 |
| Oct. | .376 | -4.649 | 1.871 | .032 |
| Nov. | .344 | -4.479 | 2.021 | .045 |
| Dec. | .320 | -4.234 | 2.139 | .046 |

Model II (Lognormal)

$$d = .525 - .086M + .013M^2 - .001M^3$$
$$m = -3.132 - .613M + .052M^2 - .001M^3$$
$$s = 2.674 - .303M + .014M^2 + .001M^3$$

RMS Error = .105

Model III (Lognormal)

$$d = .534 - .090M + .014M^2 - .001M^3 - .012 \text{ Sin } (\pi M/1.75)$$
$$m = -3.203 - .562M + .041M^2 + .134 \text{ Sin } (\pi(M-1.5)/3)$$
$$s = 2.842 - .394M + .028M^2 + .124 \text{ Sin } (\pi(M-2.5)/3)$$

RMS Error = .044

Station 22521 Honolulu, Hawaii (CONT.)

Model IV (Modified Kappa)

| | a | b | t | RMS of Fit |
|---|---|---|---|---|
| Jan. | 20 | .100 | .08 | .012 |
| Feb. | 20 | .100 | .08 | .005 |
| Mar. | 110 | .625 | .05 | .014 |
| Apr | 40 | .100 | .08 | .010 |
| May | 70 | .100 | .05 | .007 |
| Jun. | 120 | .100 | .05 | .012 |
| Jul. | 80 | .100 | .05 | .011 |
| Aug. | 70 | .100 | .05 | .011 |
| Sep. | 70 | .100 | .05 | .010 |
| Oct. | 30 | .100 | .08 | .011 |
| Nov. | 30 | .100 | .08 | .014 |
| Dec. | 20 | .100 | .08 | .005 |

Model V (Modified Kappa)

$a = -5.909 + 29.066M - 2.333M^2$

$b = .100$

$t = .096 - .014M + .001M^2$

RMS Error = 0.035

Station 26435 Nenana, Alaska

Model I (Lognormal)

| | d | m | s | RMS of Fit |
|---|---|---|---|---|
| Jan. | .565 | -4.317 | 1.649 | .018 |
| Feb. | .537 | -4.188 | 1.610 | .025 |
| Mar. | .611 | -4.450 | 1.572 | .020 |
| Apr. | .653 | -4.557 | 1.552 | .018 |
| May | .633 | -4.325 | 1.807 | .021 |
| Jun. | .477 | -3.687 | 2.004 | .028 |
| Jul. | .428 | -3.391 | 1.976 | .029 |
| Aug. | .390 | -3.446 | 1.988 | .025 |
| Sep. | .498 | -3.709 | 1.863 | .022 |
| Oct. | .491 | -4.275 | 1.624 | .024 |
| Nov. | .506 | -4.330 | 1.689 | .022 |
| Dec. | .486 | -4.613 | 1.460 | .024 |

Model II (Lognormal)

$$d = .478 + .095M - .021M^2 + .001M^3$$
$$m = -4.011 - .388M + .109M^2 - .007M^3$$
$$s = 1.653 - .085M + .034M^2 - .002M^3$$

RMS Error = .126

Model III (Lognormal)

$$d = .578 + .027M - .009M^2 - .065 \sin(\pi(M-.5)/3)$$
$$m = -4.341 - .166M + .066M^2 - .004M^3 + .254 \sin(\pi(M-.5)/3)$$
$$s = 1.705 - .085M + .031M^2 - .002M^3 - .038 \sin(\pi(M-1)/3)$$

RMS Error = .034

Station 26435 Nenana, Alaska (CONT.)

Model IV (Modified Kappa)

| | a | b | t | RMS of Fit |
|---|---|---|---|---|
| Jan. | 50 | .100 | .08 | .009 |
| Feb. | 50 | .100 | .08 | .009 |
| Mar. | 120 | .100 | .08 | .011 |
| Apr. | 120 | .100 | .05 | .010 |
| May | 60 | .100 | .05 | .004 |
| Jun. | 100 | .450 | .08 | .012 |
| Jul. | 90 | .450 | .11 | .015 |
| Aug. | 70 | .450 | .11 | .012 |
| Sep. | 30 | .100 | .11 | .015 |
| Oct. | 70 | .100 | .11 | .009 |
| Nov. | 110 | .100 | .11 | .010 |
| Dec. | 110 | .100 | .11 | .009 |

Model V (Modified Kappa)

$a = 72.27 + 1.404M + .005M^2$

$b = -.099 + .116M - .0087M^2$

$t = .070$

RMS Error = 0.045

Station 33123 Tripoli, Libya

Model I (Lognormal)

| | d | m | s | RMS of Fit |
|---|---|---|---|---|
| Jan. | .565 | -3.492 | 2.150 | .027 |
| Feb. | .704 | -3.793 | 2.086 | .017 |
| Mar. | .782 | -4.136 | 2.109 | .016 |
| Apr. | .794 | -4.623 | 1.910 | .020 |
| May | .842 | -5.051 | 1.614 | .013 |
| Jun. | .900 | -5.517 | 1.076 | .007 |
| Jul. | .975 | -5.358 | 1.128 | .002 |
| Aug. | .972 | -4.956 | 1.495 | .002 |
| Sep. | .853 | -4.627 | 1.957 | .017 |
| Oct. | .630 | -4.045 | 2.129 | .025 |
| Nov. | .590 | -3.607 | 2.290 | .026 |
| Dec. | .545 | -3.040 | 2.253 | .026 |

Model II (Lognormal)

$$d = .540 + .088M - .001M^2 - .001M^3$$
$$m = -3.024 - .565M + .023M^2 + .002M^3$$
$$s = 2.452 - .257M + .011M^2 + .001M^3$$

RMS Error = .074

Model III (Lognormal)

$$d = .301 + .204M - .019M^2 + .070\ \mathrm{Sin}\ (\pi(M-.5)/3)$$
$$m = -2.876 - .531M + .003M^2 + .004M^3 + .213\ \mathrm{Sin}\ (\pi(M-1.5)/3)$$
$$s = 2.810 - .425M + .031M^2 + .334\ \mathrm{Sin}\ (\pi(M-2)/3)$$

RMS Error = .024

Station 33123 Tripoli, Libya (CONT.)

Model IV (Modified Kappa)

| | a | b | t | RMS of Fit |
|---|---|---|---|---|
| Jan. | 60 | .275 | .08 | .018 |
| Feb. | 120 | .275 | .05 | .009 |
| Mar. | 120 | .275 | .02 | .014 |
| Apr. | 110 | .100 | .02 | .007 |
| May | 120 | .100 | .02 | .023 |
| Jun. | 120 | .100 | .02 | .036 |
| Jul. | 120 | .100 | .02 | .040 |
| Aug. | 120 | .100 | .02 | .038 |
| Sep. | 120 | .100 | .02 | .020 |
| Oct. | 40 | .100 | .05 | .009 |
| Nov. | 100 | .625 | .05 | .010 |
| Dec. | 120 | .800 | .08 | .015 |

Model V (Modified Kappa)

$a = 78.409 + 11.941M - .927M^2$

$b = .569 - .187M + .0164M^2$

$t = .094 - .026M + .002M^2$

RMS Error = 0.022

Station 41108 Saigon, Vietnam

Model I (Lognormal)

| | d | m | s | RMS of Fit |
|---|---|---|---|---|
| Jan. | .859 | -4.329 | 1.907 | .009 |
| Feb. | .961 | -4.371 | 1.836 | .003 |
| Mar. | .879 | -4.104 | 1.837 | .005 |
| Apr. | .714 | -4.061 | 2.033 | .016 |
| May | .317 | -2.060 | 2.002 | .034 |
| Jun. | .167 | -2.027 | 2.134 | .047 |
| Jul. | .140 | -2.010 | 2.123 | .044 |
| Aug. | .129 | -1.890 | 2.041 | .044 |
| Sep. | .100 | -1.924 | 2.078 | .040 |
| Oct. | .207 | -2.021 | 2.066 | .033 |
| Nov. | .500 | -3.066 | 2.248 | .026 |
| Dec. | .755 | -3.428 | 2.129 | .014 |

Model II (Lognormal)

$$d = .889 + .099M - .058M^2 + .004M^3$$
$$m = -4.667 + .019M + .117M^2 - .009M^3$$
$$s = 1.813 + .031M + .003M^2$$

RMS Error = .224

Model III (Lognormal)

$$d = .834 + .139M - .067M^2 + .005M^3 + .103 \text{ Sin } (\pi(M-1.5)/3)$$
$$m = -4.511 - .094M + .141M^2 - .011M^3 - .291 \text{ Sin } (\pi(M-1.5)/3)$$
$$s = 1.858 - .002M + .010M^2 - .001M^3 - .084 \text{ Sin } (\pi(M-1.5)/3)$$

RMS Error = .050

Station 41103 Saigon, Vietnam (CONT.)

Model IV (Modified Kappa)

| | a | b | t | RMS of Fit |
|---|---|---|---|---|
| Jan. | 120 | .100 | .02 | .007 |
| Feb. | 120 | .100 | .02 | .033 |
| Mar. | 120 | .100 | .02 | .013 |
| Apr. | 80 | .100 | .05 | .011 |
| May | 90 | 1.325 | .17 | .025 |
| Jun. | 30 | 1.150 | .23 | .027 |
| Jul. | 50 | 1.500 | .23 | .025 |
| Aug. | 50 | 1.675 | .23 | .035 |
| Sep. | 40 | 1.675 | .23 | .035 |
| Oct. | 20 | .800 | .23 | .020 |
| Nov. | 120 | .975 | .08 | .010 |
| Dec. | 50 | .100 | .05 | .016 |

Model V (Modified Kappa)

$$a = 164.318 - 27.585M + 1.646M^2$$

$$b = -.542 + .607M - .045M^2$$

$$t = -.079 + .091M - .0066M^2$$

RMS Error = 0.116

Station 45715 Shemya, Alaska

Model I (Lognormal)

| | d | m | s | RMS of Fit |
|---|---|---|---|---|
| Jan. | .028 | -3.716 | 1.770 | .035 |
| Feb. | .017 | -3.986 | 1.731 | .034 |
| Mar. | .050 | -3.992 | 1.806 | ,041 |
| Apr. | .100 | -4.217 | 1.820 | .005 |
| May | .124 | -4.217 | 1.895 | .043 |
| Jun. | .114 | -4.491 | 1.786 | .053 |
| Jul. | .105 | -4.161 | 2.031 | .055 |
| Aug. | .147 | -4.018 | 1.966 | .042 |
| Sep. | .181 | -3.726 | 1.937 | .003 |
| Oct. | .076 | -3.426 | 1.890 | .027 |
| Nov. | .042 | -3.449 | 1.849 | .030 |
| Dec. | .039 | -3.728 | 1.879 | .043 |

Model II (Lognormal)

$$d = -.004 + .019M + .002M^2$$
$$m = -3.007 - .654M + .101M^2 - .004M^3$$
$$s = 1.726 - .006M + .007M^2 - .001M^3$$

RMS Error = .240

Model III (Lognormal)

$$d = -.010 + .021M + .002M^2 - .028 \sin(\pi(M-1.5)/2)$$
$$m = -2.746 - .795M + .122M^2 - .005M^3 - .192 \sin(\pi(M+.5)/3)$$
$$s = 1.718 + .010M + .007M^2 - .001M^3 - .040 \sin(\pi(M-1.5)/2)$$

RMS Error = .045

Station 45715 Shemya, Alaska (CONT.)

Model IV (Modified Kappa)

| | a | b | t | RMS of Fit |
|---|---|---|---|---|
| Jan. | 60 | .275 | .23 | .034 |
| Feb. | 120 | .275 | .23 | .028 |
| Mar. | 110 | .275 | .20 | .017 |
| Apr. | 20 | .100 | .20 | .017 |
| May | 50 | .275 | .14 | .015 |
| Jun. | 20 | .100 | .14 | .008 |
| Jul. | 40 | .275 | .14 | .011 |
| Aug. | 30 | .275 | .14 | .014 |
| Sep. | 40 | .275 | .17 | .016 |
| Oct. | 20 | .275 | .23 | .030 |
| Nov. | 80 | .450 | .23 | .041 |
| Dec. | 50 | .275 | .23 | .013 |

Model V (Modified Kappa)

$$a = 120.455 - 22.273M + 1.434M^2$$
$$b = .303 - .035M + .0034M^2$$
$$t = .287 - .042M + .0033M^2$$

RMS Error = 0.033

Station 93780 Christchurch, New Zealand

Model I (Lognormal)

| | d | m | s | RMS of Fit |
|---|---|---|---|---|
| Jan. | .559 | -3.471 | 1.984 | .016 |
| Feb. | .591 | -3.460 | 1.972 | .016 |
| Mar. | .571 | -3.175 | 1.926 | .002 |
| Apr. | .573 | -3.145 | 1.961 | .014 |
| May | .527 | -3.225 | 1.998 | .018 |
| Jun. | .587 | -3.368 | 1.970 | .001 |
| Jul. | .500 | -3.165 | 1.940 | .002 |
| Aug. | .595 | -3.261 | 1.995 | .015 |
| Sep. | .567 | -3.407 | 1.814 | .015 |
| Oct. | .616 | -3.699 | 1.877 | .018 |
| Nov. | .590 | -2.910 | 1.981 | .014 |
| Dec. | .624 | -3.355 | 2.044 | .018 |

Model II (Lognormal)

$$d = .577 - .010M + .001M^2$$

$$m = -3.727 + .246M - .037M^2 + .002M^3$$

$$s = 1.885 + .079M - .017M^2 + .001M^3$$

RMS Error = .132

Model III (Lognormal)

$$d = .606 - .028M + .004M^2 + .023 \text{ Sin } (\pi(M-1.5))$$

$$m = -3.631 + .184M - .025M^2 + .001M^3 - .094 \text{ Sin } (\pi(M-1\ \ )/2)$$

$$s = 1.912 + .059M - .013M^2 + .001M^3 - .051 \text{ Sin } (\pi(M-1.5)/3)$$

RMS Error = .019

Station 93780 Christchurch, New Zealand (CONT.)

Model IV (Modified Kappa)

| | a | b | t | RMS of Fit |
|---|---|---|---|---|
| Jan. | 60 | .275 | .08 | .007 |
| Feb. | 80 | .275 | .08 | .009 |
| Mar. | 100 | .450 | .08 | .009 |
| Apr. | 80 | .450 | .08 | .010 |
| May | 60 | .450 | .08 | .013 |
| Jun. | 70 | .275 | .08 | .010 |
| Jul. | 40 | .275 | .11 | .012 |
| Aug. | 60 | .275 | .08 | .008 |
| Sep. | 60 | .275 | .08 | .011 |
| Oct. | 40 | .100 | .08 | .012 |
| Nov. | 70 | .450 | .08 | .008 |
| Dec. | 120 | .625 | .05 | .013 |

Model V (Modified Kappa)

$a = 100.455 - 13.177M + 1.019M^2$

$b = .430 - .045M + .0039M^2$

$t = .080$

RMS Error = .021

## PART II "EXTREME VALUES" OF DAILY PRECIPITATION

### 1. Models of "Extreme Values" for Precipitation

Two sets of models were developed. The first set consists of twelve models, one for each month of the year. The models are of the form

$$G(x) = exp\ (-exp(-((x-\mu)/\sigma)))$$

where $\mu$ and $\sigma$ are constants with $\sigma > 0$. $G(x)$ is the probability that for the month in question, the maximum (extreme) daily rainfall will be less than or equal to x. Section 2 gives the rationale for the selection of this model and discusses the methods used to get the estimates m and s for $\mu$ and $\sigma$ respectively. Section 6 gives the tabled values of m and s for the stations involved, and also an estimate of the accurary of the model.

The second set of models are valid for any month of the year. The same form is used as for the models for the individual months. However, m and s do not appear explicitly. Instead m and s are each either a polynomial in M, or a polynomial in M plus a sine term, where M represents the month of interest. For May M = 5, while for September M = 9. Section 3 gives further details on the derivation of the models.

### 2. Rationale for the Models Used

Let X be the amount of rain in a day selected at random from a specified month. Then X will be a random variable with some probability density function f(x). In general, if we have n observations $X_1, X_2, \ldots, X_n$ on a random variable X, and $X_{(i)}$ is the ith smallest of the n observations, then the cumulative distribution function of $X_{(n)}$ is given by

$$G(x) = [F(x)]^n$$

where F(x) is the cumulative distribution function of X. It is frequently the case that F(x) is not known. However, if n is large, for functions f(x) where X is unbounded and F(x) decreases at least as rapidly as the exponential function, then G(x) has the limiting form

$$G(x) = exp\ (-exp(-((x-\mu)/\sigma)))$$

where $\mu$ and $\sigma$ are constants with $\sigma > 0$. This distribution is sometimes called the "extreme value" distribution.

The parameters $\mu$ and $\sigma$ may be estimated from data by several methods. It can be shown that the maximum likelihood estimate of $\sigma$ is given by the solution of the following equation:

$$\hat{\sigma} - \bar{x} + \left[\sum_{i=1}^{n} x_i exp(-x_i/\hat{\sigma})\right] \left[\sum_{i=1}^{n} exp(-x_i/\hat{\sigma})\right]^{-1} = 0.$$

The equation may be solved for $\hat{\sigma}$ by an iterative procedure. The maximum likelihood estimate of $\hat{\mu}$ is then obtained from

$$\hat{\mu} = \left[ -\hat{\sigma} \; \ell n \; \frac{1}{n} \sum_{i=1}^{n} \; exp(-x_i/\hat{\sigma}) \right] .$$

Another method of obtaining values for $\mu$ and $\sigma$ is by the direct regression of the empirical cumulative distribution function on the model cumulative distribution.

Both methods were used in the study. The second method resulted in superior fits. Instead of the empirical cumulative distribution, the expected value of the cumulative distribution for the ith order statistic, $i/(n+1)$ was used as the dependent variable, and non-linear regression techniques were used. The values for $\mu$ and $\sigma$ chosen were those which minimized

$$\sum_{i=1}^{n} \left[ i/(n+1) - F(x_{(i)};\mu,\sigma) \right]^2$$

where $F(x_{(i)};\mu,\sigma) = exp(-exp(-((x_{(i)}-\mu)/\sigma)))$.

3. Specification of the Models

Both sets of models use the form

$$G(x) = exp(-exp(-((x-\mu)/\sigma)))$$

where $\mu$ and $\sigma$ are constants, and $G(x)$ is the probability that the maximum (extreme) daily rainfall for a specified month will be less than or equal to x. We use m and s for the estimates of $\mu$ and $\sigma$ respectively. For the first set of 12 models (one for each month) the values for m and s are given in Section 7. We shall call this MODEL I.

The second set of models are each valid for an arbitrary month. Four of these models were developed. We shall call them Models II, III, IV and V. In each case, both m and s are functions of the month M. For Model II, m and s are each a quadratic polynomial in M, while for Model III they are cubics. Models IV and V are quadratics and cubics respectively with an additional sine term.

For model II, we have

$$m = m_0 + m_1 M + m_2 M^2$$

$$s = s_0 + s_1 M + s_2 M^2$$

For model III, we have

$$m = m_0 + m_1 M + m_2 M^2 + m_3 M^3$$

$$s = s_0 + s_1 M + s_2 M^2 + s_3 M^3 .$$

Values for $m_0$, $m_1$, $m_2$, $s_0$, $s_1$, $s_2$ for Model II are given under Model II in Section

6. Values for $m_0$, $m_1$, $m_2$, $m_3$, $s_0$, $s_1$, $s_2$, $s_3$ for Model III are given under Model III in the same section. The tabulated values were obtained from regression analyses.

Models IV and V were obtained by examining the residuals from Models II and III and adding an appropriate sine term to the expressions for m and s. New regressions were then run. For model IV we have

$$m = m_0 + m_1 M + m_2 M^2 + m_3 \sin c_m$$

$$s = s_0 + s_1 M + s_2 M^2 + s_3 \sin c_s \ .$$

For model V, we have

$$m = m_0 + m_1 M + m_2 M^2 + m_3 M^3 + m_4 \sin c_m$$

$$s = s_0 + s_1 M + s_2 M^2 + s_3 M^3 + s_4 \sin c_s \ .$$

Values for the $m_i$'s, $s_i$'s, $c_m$ and $c_s$ are given in Section 6 under Models IV and V. The arguments of the sin functions were obtained from an inspection of the residuals.

## 4. Goodness of Fit of the Models

The measure of "goodness" used was the root mean square of the residuals from the regressions. *These measures are tabled in Section 6. Choice of a model should* not be made without first referring to the table. As may be observed, fits for models II, III, IV and V were inferior to the fits of Model I and in many cases substantially inferior. This should be expected since one expression is used for all 12 months while Model I has a separate expression for each month. Models III and V give better fits than Models II and IV respectively, and Models IV and V give better fits than Models II and III respectively. In general, the models with the most terms (or expressions) give the best fits.

The advantage, of course, of Models II, III, IV and V is that in each case a single expression may be used for all months.

It should be noted that in many cases the "extreme value" model does not provide as good a fit to the data as is desired. Thus, more work is needed on models to replace the "extreme value" model.

## 5. Use of the Models

The following is an example of the use of the models.

Problem: What is the probability that the maximum daily amount of rainfall in Honolulu in February will be less than 2 inches?

Solution: The model is

$$G(x) = \exp(-\exp(-((x-\mu)/\sigma)))$$

Using Model I, our estimates of $\mu$ and $\sigma$ are m = .580 and s = .662.

Substituting these values in G(x), we obtain a probability of .89.

Using Model II, $m = 1.202 - .331M + .025M^2$

$$s = 1.772 - .463M = .032M^2$$

where $M = 2$. Substituting, we have $m = .640$, $s = .974$ and the estimated probability is .78.

6. Tables of Models for "Extreme Values" for Precipitation

Station 12867 Patrick AFB

Model I

| | m | s | RMS |
|---|---|---|---|
| Jan. | 0.553 | 0.833 | 0.034 |
| Feb. | 0.913 | 0.770 | 0.051 |
| Mar | 0.780 | 0.757 | 0.030 |
| Apr. | 0.584 | 0.563 | 0.030 |
| May | 0.885 | 0.787 | 0.025 |
| Jun. | 1.239 | 0.964 | 0.024 |
| Jul. | 1.018 | 0.404 | 0.033 |
| Aug. | 1.276 | 0.562 | 0.039 |
| Sep. | 1.828 | 1.034 | 0.038 |
| Oct. | 1.201 | 1.155 | 0.033 |
| Nov. | 0.653 | 0.858 | 0.048 |
| Dec. | 0.524 | 0.479 | 0.019 |

Model II

$$m = .162 + .293M - .0206M^2$$
$$s = .750 + .0008M - .0002M^2$$

RMS Error = .121

Model III

$$m = .975 - .335M + .0956M^2 - .006M^3$$
$$s = 1.173 - .326M + .0605M^2 - .0031M^3$$

RMS Error = .099

Model IV

$$m = .057 + .309M - .021M^2 - .270 \sin(\pi(M-2.5)/2)$$
$$s = .653 + .016M + .000M^2 - .251 \sin(\pi(M-2.5)/2)$$

RMS Error = .092

Model V

$$m = 1.337 - .565M + .136M^2 - .008M^3 - .256 \sin(\pi(M-3.5)/1.5)$$
$$s = 1.194 - .339M + .063M^2 - .003M^3 + .027 \sin(\pi(M-1.5)/1.5)$$

RMS Error = .075

Station 13802 Scott AFB

Model I

| | m | s | RMS |
|---|---|---|---|
| Jan. | 0.517 | 0.462 | 0.050 |
| Feb. | 0.613 | 0.498 | 0.026 |
| Mar. | 0.752 | 0.421 | 0.023 |
| Apr. | 0.840 | 0.517 | 0.043 |
| May | 0.955 | 0.591 | 0.032 |
| Jun. | 1.095 | 0.714 | 0.036 |
| Jul. | 0.897 | 0.734 | 0.022 |
| Aug. | 0.938 | 0.806 | 0.027 |
| Sep. | 0.945 | 0.754 | 0.031 |
| Oct. | 0.697 | 0.569 | 0.015 |
| Nov. | 0.722 | 0.462 | 0.028 |
| Dec. | 0.663 | 0.603 | 0.040 |

Model II

$$m = .328 + .188M - .014M^2$$
$$s = .279 + .109M - .007M^2$$

RMS Error = .050

Model III

$$m = .244 + .253M - .026M^2 + .001M^3$$
$$s = .430 - .007M + .014M^2 - .001M^3$$

RMS Error = .049

Model IV

$$m = .328 + .188M - .014M^2 + .001\ \mathrm{Sin}(\pi(M-1.5))$$
$$s = .214 + .130M - .009M^2 - .095\ \mathrm{Sin}(\pi(M-2.5)/3)$$

RMS Error = .046

Model V

$$m = .239 + .256M - .026M^2 + .001M^3 - .004\ \mathrm{Sin}(\pi(M-1.5))$$
$$s = .310 + .058M - .005M^2 - .001M^3 - .088\ \mathrm{Sin}(\pi(M-2.5/3)$$

RMS Error = .045

## Station 14601 Bangor, Maine

### Model I

| | m | s | RMS |
|---|---|---|---|
| Jan. | 0.811 | 0.371 | 0.031 |
| Feb. | 0.916 | 0.285 | 0.049 |
| Mar. | 0.762 | 0.350 | 0.034 |
| Apr. | 0.772 | 0.258 | 0.036 |
| May | 0.594 | 0.367 | 0.032 |
| Jun. | 0.825 | 0.427 | 0.043 |
| Jul. | 0.948 | 0.563 | 0.068 |
| Aug. | 0.607 | 0.466 | 0.052 |
| Sep. | 0.826 | 0.505 | 0.046 |
| Oct. | 1.273 | 0.494 | 0.040 |
| Nov. | 1.192 | 0.606 | 0.044 |
| Dec. | 0.869 | 0.554 | 0.048 |

### Model II

$$m = .879 - .050M + .006M^2$$

$$s = .282 - .019M + .001M^2$$

RMS Error = .104

### Model III

$$m = 1.093 - .215M + .036M^2 - .002M^3$$

$$s = .405 - .076M + .018M^2 - .001M^3$$

RMS Error = .101

### Model IV

$$m = .914 - .065M + .007M^2 - .179 \sin(\pi(M-3.5)/2)$$

$$s = .272 + .021M + .001M^2 - .037 \sin(\pi(M-1.5))$$

RMS Error = .082

### Model V

$$m = 1.128 - .230M + .038M^2 - .002M^3 + .179 \sin(\pi(M-1.5)/2)$$

$$s = .364 - .050M + .014M^2 - .001M^3 - .032 \sin(\pi(M-1.5))$$

RMS Error = .079

Station 14702 Bedford, Massachussets

Model I

| | m | s | RMS |
|---|---|---|---|
| Jan. | 0.914 | 0.486 | 0.036 |
| Feb. | 0.902 | 0.462 | 0.024 |
| Mar. | 1.035 | 0.521 | 0.024 |
| Apr. | 0.959 | 0.445 | 0.050 |
| May | 0.787 | 0.449 | 0.051 |
| Jun. | 0.739 | 0.388 | 0.031 |
| Jul. | 0.861 | 0.624 | 0.023 |
| Aug. | 0.866 | 0.656 | 0.046 |
| Sep. | 0.788 | 1.234 | 0.055 |
| Oct. | 0.993 | 0.542 | 0.052 |
| Nov. | 1.386 | 0.493 | 0.051 |
| Dec. | 0.970 | 0.530 | 0.050 |

Model II

$m = 1.060 - .080M + .007M^2$

$s = .324 + .069M - .004M^2$

RMS Error = .087

Model III

$m = 1.003 - .037M - .001M^2$

$s = .755 - .264M + .058M^2 - .003M^3$

RMS Error = .085

Model IV

$m = 1.098 - .086M + .007M^2 + .100\ \mathrm{Sin}(\pi(M-2.5)/2)$

$s = .185 + .100M - .005M^2 - .199\ \mathrm{Sin}(\pi(M-3.5)/3.5)$

RMS Error = .075

Model V

$m = 1.186 - .151M + .019M^2 - .001M^3 + .112\ \mathrm{Sin}(\pi(M-2.5)/2)$

$s = .814 - .315M + .068M^2 - .004M^3 + .101\ \mathrm{Sin}(\pi(M-1.5)/2.5)$

RMS Error = .076

Station 22521 Honolulu, Hawaii

Model I

| | m | s | RMS |
|---|---|---|---|
| Jan. | 0.947 | 1.585 | 0.048 |
| Feb. | 0.580 | 0.662 | 0.040 |
| Mar. | 0.444 | 0.692 | 0.068 |
| Apr. | 0.233 | 0.349 | 0.071 |
| May | 0.189 | 0.321 | 0.088 |
| Jun. | 0.065 | 0.099 | 0.072 |
| Jul. | 0.148 | 0.238 | 0.063 |
| Aug. | 0.122 | 0.163 | 0.075 |
| Sep. | 0.205 | 0.260 | 0.044 |
| Oct. | 0.389 | 0.496 | 0.042 |
| Nov. | 0.558 | 0.145 | 0.056 |
| Dec. | 0.750 | 1.020 | 0.046 |

Model II

$$m = 1.202 - .331M + .025M^2$$
$$s = 1.772 - .463M = .032M^2$$

RMS Error = .091

Model III

$$m = 1.271 - .384M + .035M^2 - .001M^3$$
$$s = 1.926 - .582M + .054M^2 - .001M^3$$

RMS Error = .088

Model IV

$$m = 1.195 - .330M + .025M^2 - .025 \operatorname{Sin}(\pi(M-1.5))$$
$$s = 1.767 - .462M + .032M^2 - .015 \operatorname{Sin}(\pi(M-1.5))$$

RMS Error = .085

Model V

$$m = 1.243 - .366M + .031M^2 - .022 \operatorname{Sin}(\pi(M-1.5))$$
$$s = 1.916 - .576M + .053M^2 - .001M^3 - .008 \operatorname{Sin}(\pi(M-1.5))$$

RMS Error = .082

Station 26435 Nenana, Alaska

Model I

| | m | s | RMS |
|---|---|---|---|
| Jan. | 0.120 | 0.106 | 0.039 |
| Feb. | 0.103 | 0.114 | 0.075 |
| Mar. | 0.081 | 0.046 | 0.066 |
| Apr. | 0.075 | 0.073 | 0.046 |
| May | 0.180 | 0.117 | 0.044 |
| Jun. | 0.428 | 0.267 | 0.026 |
| Jul. | 0.415 | 0.244 | 0.048 |
| Aug. | 0.444 | 0.314 | 0.055 |
| Sep. | 0.272 | 0.188 | 0.043 |
| Oct. | 0.124 | 0.078 | 0.032 |
| Nov. | 0.103 | 0.086 | 0.023 |
| Dec. | 0.124 | 0.114 | 0.066 |

Model II

$$m = -.105 + .120M - .009M^2$$
$$s = -.015 + .061M - .004M^2$$

RMS Error = .232

Model III

$$m = .083 - .025M + .018M^2 - .001M^3$$
$$s = .105 - .031M + .013M^2 - .001M^3$$

RMS Error = .220

Model IV

$$m = -.077 + .103M - .007M^2 - .146 \, \mathrm{Sin}(\pi(M-2\quad)/3.5)$$
$$s = -.076 + .081M - .006M^2 - .089 \, \mathrm{Sin}(\pi(M-2.5)/3)$$

RMS Error = .077

Model V

$$m = -.096 + .072M + .004M^2 - .001M^3 - .132 \, \mathrm{Sin}(\pi(M-2.5)/3)$$
$$s = -.009 + .031M + .004M^2 - .001M^3 - .084 \, \mathrm{Sin}(\pi(M-2.5)/3)$$

RMS Error = .071

Station 33123 Tripoli, Libya

Model I

| | m | s | RMS |
|---|---|---|---|
| Jan. | 0.434 | 0.279 | 0.053 |
| Feb. | 0.242 | 0.475 | 0.027 |
| Mar. | 0.167 | 0.253 | 0.034 |
| Apr. | 0.089 | 0.214 | 0.032 |
| May | -0.008 | 0.123 | 0.065 |
| Jun. | -0.010 | 0.028 | 0.052 |
| Jul. | -0.076 | 0.045 | 0.016 |
| Aug. | -0.034 | 0.038 | 0.015 |
| Sep. | -0.394 | 0.958 | 0.040 |
| Oct. | 0.346 | 0.387 | 0.048 |
| Nov. | 0.679 | 0.689 | 0.035 |
| Dec. | 0.712 | 0.465 | 0.056 |

Model II

$$m = .777 - .292M + .024M^2$$
$$s = .468 - .112M + .011M^2$$

RMS Error = .135

Model III

$$m = .474 - .058M - .019M^2 + .002M^3$$
$$s = .723 - .309M + .047M^2 - .002M^3$$

RMS Error = .126

Model IV

$$m = .856 - .304M + .024M^2 + .146\ \mathrm{Sin}(\pi(M-3.5)/3)$$
$$s = .243 - .030M + .005M^2 - .101\ \mathrm{Sin}(\pi(M-4.5)/6)$$

RMS Error = .115

Model V

$$m = .407 - .014M - .027M^2 + .003M^3 - .106\ \mathrm{Sin}(\pi(M-2.5)/1.5)$$
$$s = 1.012 - .627M + .116M^2 - .006M^3 + .266\ \mathrm{Sin}(\pi(M-1.5)/4)$$

RMS Error = .133

Station 41108 Saigon, Vietnam

Model I

| | m | s | RMS |
|---|---|---|---|
| Jan. | -0.435 | 0.828 | 0.055 |
| Feb. | -0.081 | 0.116 | 0.018 |
| Mar. | -0.074 | 0.457 | 0.089 |
| Apr. | 0.234 | 0.592 | 0.081 |
| May | 1.562 | 2.184 | 0.078 |
| Jun. | 2.201 | 0.812 | 0.081 |
| Jul. | 1.893 | 0.787 | 0.044 |
| Aug. | 2.004 | 1.220 | 0.077 |
| Sep. | 2.304 | 0.681 | 0.096 |
| Oct. | 2.073 | 1.194 | 0.041 |
| Nov. | 0.761 | 0.468 | 0.052 |
| Dec. | 0.345 | 0.274 | 0.015 |

Model II

$$m = -1.991 + 1.044M - .069M^2$$

$$s = .053 + .322M - .025M^2$$

RMS Error = .210

Model III

$$m = .579 - .047M + .133M^2 - .010M^3$$

$$s = .381 + .069M + .022M^2 - .002M^3$$

RMS Error = .169

Model IV

$$m = 2.202 + 1.077M - .069M^2 - .546 \sin(\pi(M-2.5)/2)$$

$$s = .025 + .326M - .025M^2 - .102 \sin(\pi(M-2.5)/1.5)$$

RMS Error = .158

Model V

$$m = -1.268 + .385M + .058M^2 - .007M^3 - .421 \sin(\pi(M-2.5)/2)$$

$$s = .235 + .165M + .004M^2 - .002M^3 - .232 \sin(\pi(M-1.5))$$

RMS Error = .154

Station 45715 Shemya, Alaska

Model I

| | m | s | RMS |
|---|---|---|---|
| Jan. | 0.397 | 0.213 | 0.039 |
| Feb. | 0.301 | 0.197 | 0.051 |
| Mar. | 0.304 | 0.213 | 0.042 |
| Apr. | 0.326 | 0.292 | 0.062 |
| May | 0.390 | 0.233 | 0.026 |
| Jun. | 0.325 | 0.356 | 0.045 |
| Jul. | 0.530 | 0.335 | 0.042 |
| Aug. | 0.679 | 0.355 | 0.043 |
| Sep. | 0.588 | 0.443 | 0.038 |
| Oct. | 0.628 | 0.340 | 0.053 |
| Nov. | 0.542 | 0.393 | 0.030 |
| Dec. | 0.402 | 0.296 | 0.034 |

Model II

$m = .215 + .058M - .003M^2$

$s = .120 + .050M - .003M^2$

RMS Error = .108

Model III

$m = .584 - .227M + .050M^2 - .003M^3$

$s = .235 - .038M + .014M^2 - .001M^3$

RMS Error = .059

Model IV

$m = .299 + .045M - .003M^2 - .145\ \mathrm{Sin}(\pi(M-1.5)/5)$

$s = .118 + .051M - .003M^2 - .008\ \mathrm{Sin}(\pi(M-1.5))$

RMS Error = .062

Model V

$m = .583 - .226M + .050M^2 - .003M^3 + .001\ \mathrm{Sin}(\pi(M-2.5))$

$s = .232 - .037M + .013M^2 - .001M^3 - .002\ \mathrm{Sin}(\pi(M-1.5))$

RMS Error = .059

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